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An alternative definition and use for the constraint function for rule-based methods of functional regionalisation

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Abstract. The paper addresses the issue of the constraint function and its use in methods of regional taxonomy and assessment of regional systems. It proposes an alternative definition for the constraint function for rule-based methods of functional regionalisation, such as the delineation of local labour-market areas, including alternative conceptions of the self-containment of the regions, and the trade-off between size and self-containment. An alternative constraint function has been applied in the territory of the Czech Republic as shown in the paper's illustrative example of its use from the original CURDS algorithm.

Keywords: regional taxonomy, functional regionalisation, constraint function, local labour market areas, travel-to-work areas, travel-to-work flows, Czech Republic

1 Introduction

Functional regionalisation attempts to delineate functional regions in a defined territory. Functional regions are usually understood to be the areas organised by the horizontal functional relations (flows, interactions) that are maximised within a region and minimised across its borders so that the principles of internal cohesiveness and external separation regarding spatial interactions are met (see, for instance, Farmer and Fotheringham, 2011; Fischer, 1980; Klapka et al, 2013a; Smart, 1974). A functional region is not an abstract spatial concept, but to a certain extent it can be regarded as a reflection of the spatial behaviour of individuals in a geographic space. Therefore, in this paper we try to place our findings in the context of human spatial behaviour and spatial organisation.

The functional region is conceived as a general concept while its inner structure, inner spatial flows, and interactions need not necessarily show any regular pattern, only self-containment. The concept of self-containment remains the only crucial defining characteristic of a functional region. Nodal regions, functional urban regions, daily urban systems, local labour-market areas (LLMAs), or travel-to-work areas are considered to be special instances of a general functional region that need to fulfil some specific conditions regarding, for instance, the character of the region-organising interaction or the presence of urban cores, (Klapka et al, 2013a).

In this paper we attempt to offer a methodological contribution and refinement to the delineation of LLMAs. These areas are specific instances of functional regions based on daily travel-to-work flows, and in the case presented in this paper they are organised around cores that have a mostly urban character. The remainder of the paper consists of the following sections: section 2 discusses the relationship between the spatial behaviour of the population and the concept of LLMAs. Section 3 includes a brief overview of the methods of functional regional taxonomy and addresses the issue of LLMAs. Section 4 discusses an alternative definition and use for the constraint function as applied in rule-based methods of functional regionalisation, and proposes some new possibilities for input parameters regarding the size and self-containment of LLMAs and their role in the formulation of an alternative definition for the constraint function. Section 5 shows the results of the application of the regionalisation procedure in the territory of the Czech Republic, as well as examples of operations using the newly defined constraint function. Section 6 briefly comments on the geography of the results and provides a concise comparison of the results with other methods that have been used in the Czech Republic. Section 7 highlights the contributions proposed by the paper relating to the issue under consideration, sums up the findings of the paper, and returns to more general considerations with regard to human spatial behaviour and its role in regionalisation tasks.

2 Local labour-market areas and spatial behaviour

Delineation of functional regions (LLMAs in this case) is an important part of geographical research and spatial analysis. The results of functional regionalisation have a wide range of applications (Casado-Díaz and Coombes, 2011; Coombes, 2010). LLMAs can be used for labour-market analysis, both in a spatial and in a temporal sense (structure of the labour market, unemployment rates, development trends). They can be also used for the assessment of regional disparities, and as a tool for the distribution of economic and financial subsidies to underdeveloped and structurally afflicted regions. For all these purposes LLMAs can be better than administrative units as they are functionally organised.

Casado-Díaz and Coombes (2011) point out that the delineation of LLMAs has to be based on travel-to-work data, as an examination of the potential spatial behaviour of the population in the labour market is not possible. The substance of LLMAs rests in the analysis of the spatial patterns of statistical daily travel-to-work flows. However, LLMAs, if correctly defined, can be understood as a representative spatial image or imprint of some aspects of the spatial behaviour of a population (see, for instance, Hanson and Pratt, 1992; Heldt Cassel et al, 2013). The daily travel-to-work flows are the most numerous regular daily movements for a large part of the population within a residential area—a workplace time–space prism that covers most of the nonleisure activities of individuals [for the concept of a prism see, for instance, Lenntorp (1976) or Pred (1977)]. These flows are based on two types of choices made by individuals. An individual has to decide on the location of his or her residence and of his or her workplace (Golledge and Stimson, 1997). Furthermore, an individual has to consider the friction of distance (expressed in units of length, time, or cost) between the two places. An individual has to weigh the benefits and costs of the outline situation, including the accessibility of transport, that mediates the contact between residential and workplace locations. Based on these assumptions the daily travel-to-work flows and their aggregations in the form of functional regions include a great many and variety of individual perceptions, demands, and decisions. The functionality of LLMAs is secured by their self-containment, which means that the manifestations of spatial behaviour in the form of daily travel-to-work flows should be predominantly limited to being within such a region and minimised across its borders (see relevant passages below).

LLMAs, as functional regions based on aggregations of individual spatial behaviour, are not merely simple schemes passively mediating spatial information (Bezák, 2000), but thanks to their rigorous definition they represent significant effective and functional tools for identification and analysis of selected aspects of spatial behaviour. Spatial patterns of flows between origins and destinations within LLMAs are very complex, though they manifest certain regularities (Erlebach et al, 2014). The early stages of research, using the concept of a functional region and regarding the spatial behaviour of the population, were oriented to population development in core and peripheral parts of functional regions (Klaassen and Scimeni, 1981; Van den Berg et al, 1982). These works help us to understand the principles of the stages of urban development processes, particularly suburbanisation. From the 1970s the deconcentration processes have not been limited to urban areas alone, but they can also be identified in the whole settlement system (Bezák, 2000). Berry (1976) calls these types of processes at all scales 'counterurbanisation'. Geyer and Kontuly (1993), in the theory of differential urbanisation, claim that concentration and deconcentration processes can be underway simultaneously. With respect to this, Champion (2003) used LLMAs to test differential urbanisation in Great Britain. Moreover, the motivations of individuals to migrate can be significantly different if the migration occurs within an LLMA or if it occurs across its borders. In the former instance the prevailing reason is a change of residential location; in the latter the prevailing reason is a change of workplace location.

3 Local labour-market areas: methodological foundations

Numerical methods for regional taxonomy were introduced into geography by many geographers, Spence and Taylor (1970) among them. Over the almost five decades since then several approaches to the delineation of functional regions, particularly LLMAs have appeared (Casado-Díaz and Coombes, 2011). LLMAs are based on the most frequent and most stable regular movement of the population with daily periodicity, and their delineation is a special instance of functional regional taxonomy.

The methods for the delineation of functional regions can generally be sorted into two groups: clustering methods and rule-based methods. Furthermore, several variants can be defined for each case conforming (a) either to divisive or to agglomerative procedures, (b) either to hierarchical or to nonhierarchical procedures, and (c) either to numerical or to graph-theoretical procedures. The aforementioned classification is only one of several possibilities. Alternative classifications of the regionalisation procedures are discussed, for instance, by Coombes (2000), Van der Laan and Schalke (2001), Casado-Díaz and Coombes (2011), and Farmer and Fotheringham (2011).

Out of the all possible combinations the literature favours the following approaches to functional regional taxonomy. Clustering methods have been used by Brown and Holmes (1971), Masser and Brown (1975), Masser and Scheurwater (1978; 1980), Fischer [(1980)—including the nonhierarchical variant], and Baumann et al (1983). Relatively recently these methods have been applied by Cörvers et al (2009), Krygsman et al (2009), and Mitchell and Watts (2010), who favoured the clustering methods over the rule-based methods. Graph-theoretical methods have been proposed by Nystuen and Dacey (1961), Slater (1976), Holmes and Haggett (1977), and recently applied by Karlsson and Olsson (2006).

LLMAs are delineated in most cases by the application of rule-based methods. The first attempts were presented by Smart (1974), who recommended but did not use what is currently the most frequent interaction measure:

$$\left(\frac{T_{ij}^2}{\sum_k T_{ik} \sum_k T_{kj}} + \frac{T_{ji}^2}{\sum_k T_{jk} \sum_k T_{ki}} \right),$$

where T_{ij} is the value of a flow from municipality i to municipality j, T_{ji} is the value of a flow from municipality j to municipality i, and k is the total number of basic spatial units (municipalities) in the system. Smart's measure is mathematically the most correct method of relativisation and symmetrisation of two-dimensional interaction statistical data. An overview of the interaction measures is provided by Casado-Izquierdo and Propín-Frejomil (2008). Smart's procedure was later criticised as being purely heuristic (eg, Ball, 1980; Coombes and Openshaw, 1982).

The rule-based procedure proposed by Coombes et al, (1979; 1982), particularly its second (Coombes et al, 1986) and third variants (Coombes, 2010), has gained the most attention and frequent applications. The second variant has been used to delineate LLMAs (or travel-to-work areas) according to the daily labour commuting flows in different countries: Spain (Casado-Díaz, 2000), Slovakia (Bezák, 2000; Halás et al, 2014), New Zealand (Newell and Perry, 2005; Papps and Newell, 2002), Ireland (Meredith et al, 2007), South Africa (Nel et al, 2008), Belgium (Persyn and Torfs, 2011), Poland (Gruchociak, 2012), and the Czech Republic (Klapka et al, 2013b; 2014). Some of the principles of the clustering methods have been included in the third variant of the CURDS (Centre for Urban and Regional Development Studies) algorithm (Coombes, 2010), where only the application of the constraint function has remained as the crucial rule. A comparison of the results of the clustering and rule-based methods has been carried out by Landré and Håkansson (2013) based on the example of Sweden. Watts (2013) compared the results for different algorithms in New South Wales.

The second variant of the CURDS algorithm (Coombes et al, 1986) is applied in this paper in order to test the alternative definition and use of the constraint function and in order to delineate the LLMAs of the Czech Republic. The second variant has been used for several purposes. The first of these is the comparability of the procedure and the results. Second, it can be useful to identify a set of potential regional cores, a possibility that the newest variant does not provide. Knowledge of the cores is important for further analyses of the inner structure of the resulting regions. Third, the parameters for the second variant can be set in such a manner that it actually behaves in the same way as the newest variant. The procedure itself is described in comprehensive detail by Coombes et al, (1986; page 948–952). Minor alterations in the value of the parameters that have been made in this paper are described in section 4, which addresses in detail the constraint function and its role in the regionalisation algorithm.

4 Local labour-market areas: constraint function

The constraint function introduces two important rules into the regionalisation procedure that have to be obeyed by every resulting region: the minimum size of a region and the minimum self-containment of a region with regard to daily travel-to-work flows. While the minimum self-containment is a crucial parameter and should not drop below 0.5 (more than half of the flows incident to a region should have their origins and destinations within that region), the minimum size is introduced for practical purposes and it conforms to the objectives of the research.

A terminological note should be made here. The function setting the zone design constraints (the criteria for the size and self-containment of LLMAs) was originally called the *objective function* (Coombes et al, 1986). Other works (for instance, Casado-Díaz, 2000; Papps and Newell, 2002) resorted to using the general term *function*. Coombes and Bond (2008) came up with the term *X equation*. Coombes (2010) used only the periphrasis *size and self-containment values combined into a linear spline function*. As in operations research the term 'objective function' refers to a function allowing comparisons between different solutions to a given task [for instance, in the delineation of functional regions as used by Flórez-Revuelta et al (2008)], it appears preferable to use the term *constraint function* for the solution proposed in this paper, because it sets two zone design criteria, the trade-off between them, and also decides between a valid and invalid solution.

4.1 Size of a region

The minimum size of a region can be defined by several criteria. The most general characteristic is the population of a region and that is a standard, simple, and easily accessible criterion. However, if the criteria used in the regionalisation procedure are also to be used for the definition of the size of a region, there are three options: the definition of the minimum size according to the flows $\sum_k T_{jk}$ (the sum of outgoing flows including inner flows), $\sum_k T_{kj}$ (the sum of ingoing flows including inner flows), or T_{jj} (inner flows within a region). $\sum_k T_{jk}$ is the resident working population of a region (either working in a region or travelling to work across its borders). In a simplified way it is the number of economically active persons with the reservation that it can be defined differently by individual state legislatures. $\sum_k T_{kj}$ is the working population in a region (either resident in a region or travelling to work from the outside of a region). In a simplified way it is the number of jobs in a region. T_{jj} is the resident population working locally in a region: that is, the inner flow.

The choice of criterion depends on the type of task to be solved. For general regionalisation tasks $\sum_k T_{jk}$ appears to be the most convenient as it correlates best with the population of a region. If the labour market and total economic potential of a region are to be emphasised, it is better to use $\sum_k T_{kj}$. T_{jj} flow as a combination of the two preceding criteria is a compromise between $\sum_k T_{jk}$ and $\sum_k T_{kj}$. Together with the number of economically active persons and the number of jobs it also partly includes the total self-containment of a region.

4.2 Self-containment of a region

The value of the relative self-containment of a region is a primary criterion for the definition of functional regions. In the CURDS algorithm self-containment has been defined by:

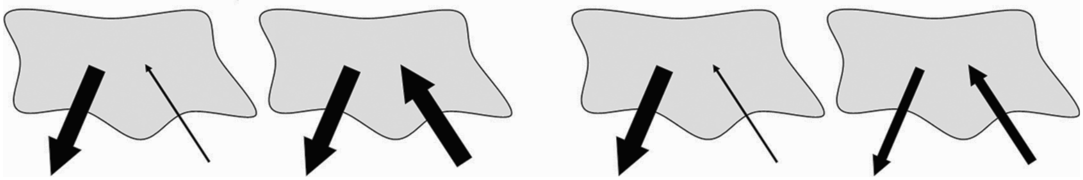
$$\frac{T_{jj}}{\max\left(\sum_k T_{jk};\sum_k T_{kj}\right)} ;$$

that is, the ratio between inner flows and whichever has the greater value between the number of economically active persons and the number of jobs. It is the smaller value of a workplace-based self-containment and residence-based self-containment and it can be called the 'index of unidirectional self-containment'. As an alternative to this formula, in this paper we propose that self-containment be defined as:

$$\frac{T_{jj}}{\left(\sum_k T_{jk} + \sum_k T_{kj}\right) - T_{jj}} ,$$

which is able to assess the self-containment of a region with regard to both flow directions crossing the borders of a region (to and from the outside). It is a more complex way of assessing self-containment and can be called the 'index of total self-containment' (note that the denominator, T_{jj} is included in both $\sum_k T_{jk}$ and $\sum_k T_{kj}$ and therefore it has to be subtracted from the summation of all flows incident to the region). Both concepts of self-containment have their advantages and disadvantages. The index of unidirectional self-containment is not able to distinguish between the case where one of the flows (to or from the outside) is high and the other is low from the case where both flows (to and from the outside) are high [figure 1(a)]. In both cases the index provides the same numerical value.

In contrast, the index of total self-containment is not able to distinguish between the case where one of the flows (to or from the outside) is high and the other is low from the case where both flows (to and from the outside) are medium-sized [figure 1(b)]. Again, in both cases the index provides the same numerical value.



(a) (b)

Figure 1. Specifics of (a) the index of unidirectional self-containment (in both instances the index has the same value; and (b) the index of total self-containment (in both instances the index has the same value.

The index of unidirectional self-containment provides significantly different values in the second model example where it works only with the greater flow crossing the border of a region. Apart from the fact that one of the flows can be minimal or zero, the disadvantage of this index rests in its inability to indicate which flow (to or from the outside) provides the numerical expression of the index. Therefore the index of total self-containment appears to be more convenient and complex and it expresses the self-containment of a region regardless of the direction of flows crossing the border of the region. In section 5 we will therefore use the index of total self-containment.

The difference between the numerical values provided by the indices of unidirectional and total self-containment is not easily determinable. It is dependent on the hierarchical level of the region, the ratio of inflows and outflows, and spatial patterns of regional systems. The analysis of various regional systems of the Czech Republic based on 2001 Census data has revealed that the value of the index of unidirectional self-containment is, on average, 0.05 higher for the resulting regions than the value of the index of total self-containment.

A note regarding both self-containment indexes and the definition of functional regions, as stated above, should be made at this point. If it is assumed that a functional region follows the principles of inner coherence and external self-containment, then the index of total self-containment offers a crucial advantage. When its value is lower than 0.66, the index of unidirectional self-containment in the regionalisation algorithm can provide results in regions that do not conform to the basic definition of a functional region, whereas the use of the index of total self-containment always provides regions conforming to that definition (ie, the value of the minimal self-containment of a region never drops below 0.5 in this latter case).

4.3 Constraint function

The constraint function is an important part of the regionalisation algorithm and it determines which regions will be maintained and will appear in the resulting regionalisation. These regions have to be a certain minimum size and have minimum self-containment defined by the constraint function. The constraint function also takes into account the trade-off between the two criteria, when smaller regions have to manifest higher self-containment and larger regions are allowed to show lower self-containment.

In the CURDS algorithm (Coombes et al, 1986) the definition of the constraint function has used the linear spline for the trade-off and four limits have been set (the upper and lower limits of size and the upper and lower limits of self-containment) for the trade-off.[(1)] For values exceeding the upper limits of size and self-containment of a region the trade-off has not been used. Papps and Newell (2002) have drawn attention to the fact that such a

[(1)] Two printing errors can be found in the original paper in the notation of the constraint function, one in the decimal place and the other in the bracket placement. However, the application of the algorithm and the results are correct. Correct notation of the constraint function can be found in Papps and Newell (2002, page 12).

constraint function does not apply to a linear spline but that the trade-off between the size and self-containment of a region is expressed by a very slightly deflected function and not by a flat line segment. Casado-Díaz (2000) has used a notation in which the trade-off has been expressed by a flat line segment and not by a single constraint function but by a notation with three conditions. Casado-Díaz (2000) can already accurately control all four limits (upper and lower) of size and self-containment of regions.

Alternative definitions for the constraint function will now be proposed. All functions respect the trade-off between size and self-containment and control accurately all four limits of the size and self-containment of regions (both upper and lower). The constraint function has the form:

$$\min\left\{1; \max\left[0; \alpha_7 + \frac{(1-\alpha_7)\left(\sum_k T_{jk} - \beta_3\right)}{\beta_4 - \beta_3}\right]\right\} \times \min\left\{1; \alpha_7 + \frac{(1-\alpha_7)}{\beta_2 - \beta_1}\left[\frac{T_{jj}}{\left(\sum_k T_{jk} + \sum_k T_{kj}\right) - T_{jj}} - \beta_1\right]\right\} \geqslant \alpha_7 ,$$

where β_1, β_2, β_3, and β_4 are the limits of the trade-off between the size and self-containment of a region (β_1 and β_2 are lower and upper limits of the self-containment; β_3 and β_4 are lower and upper limits of the size). They can be controlled without the need for demanding recalculations and each of the limits can be easily set for any logical value. The parameter α_7 ($0 < \alpha_7 < 1$) determines the measure of the deflection of the trade-off part of the function. The closer it approaches the value of 1, the more the trade-off approximates a straight line [for example, in figure 2(a) where $\alpha_7 = 0.9$]. In contrast, the closer the parameter α_7 approaches the value of 0, the more the trade-off follows a deflected curve [for example, in figure 2(b) where $\alpha_7 = 0.1$].(2)

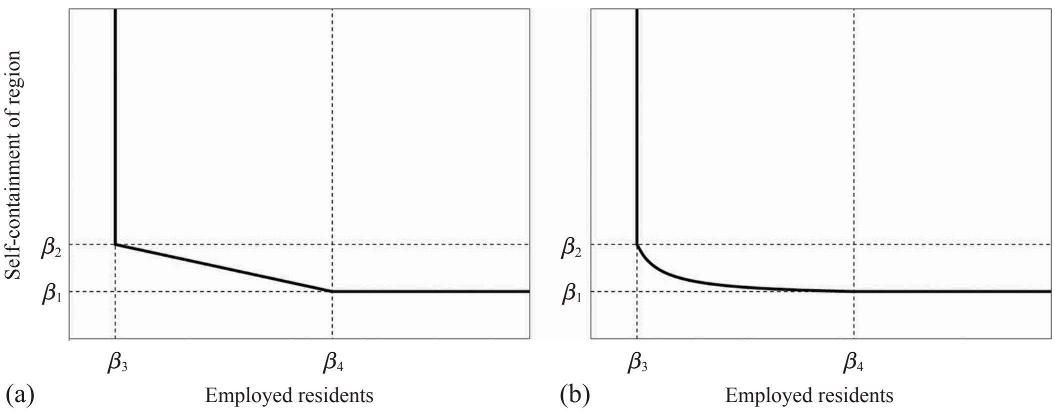


Figure 2. Angled constraint function with (a) an almost linear trade-off; (b) a deflected trade-off.

In these constraint functions the trade-off appears only in a certain interval between the upper and lower limits of the size and self-containment. In fact, the constraint function consists of three linked lines. A much more elegant solution would rest in the general validity of the trade-off between the size and self-containment (for instance, as an inverse proportion).

(2) Parameter α_7 is not used in the original algorithm by Coomber et al (1986); it is comprised in the objective functions proposed by this paper.

The constraint function would be expressed by a single smooth curve. It is possible to formulate such a function in the form:

$$\frac{T_{jj}}{\left(\sum_k T_{jk} + \sum_k T_{kj}\right) - T_{jj}} - \frac{\alpha_8(\beta_4 - \beta_3)(\beta_2 - \beta_1)}{\left|\max\left(\beta_3 + 1; \sum_k T_{jk}\right) - \beta_3\right|} \geqslant \beta_1 .$$

This function will be called the continuous constraint function and its shape is presented in figure 3. The value of parameter determines where the continuous constraint function intersects the original function from figure 2(a). Thus in figure 3 $\alpha_8 = 0.09$ and the continuous constraint function intersects the original function at $\frac{1}{10}$ and $\frac{9}{10}$ of the section β_1–β_2 and at $\frac{1}{10}$ and $\frac{9}{10}$ of the section β_3–β_4 [generally the continuous constraint function intersects the original function at $1/(100\alpha_8 + 1)$ and $100\alpha_8/(100\alpha_8 + 1)$ of given sections].(3) The continuous constraint function is constructed in order to approach asymptotically the lower limits of the size and self-containment of a region.

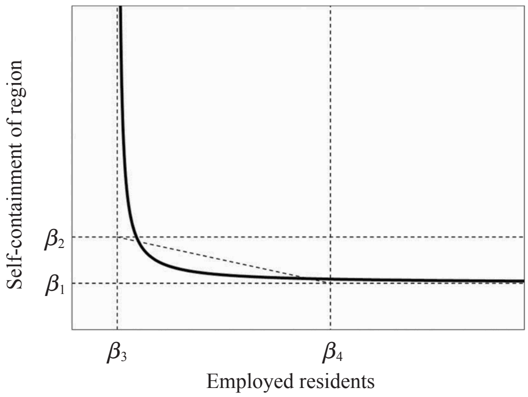


Figure 3. Continuous constraint function.

In both defined types of the constraint function three size (SZ) criteria:

$$\mathrm{SZ}_1 = \sum_k T_{jk}, \qquad \mathrm{SZ}_2 = \sum_k T_{kj}, \quad \mathrm{SZ}_3 = T_{jj} ,$$

and two self-containment (SC) indices:

$$\mathrm{SC}_1 = \frac{T_{jj}}{\max\left(\sum_k T_{jk}; \sum_k T_{kj}\right)}, \qquad \mathrm{SC}_2 = \frac{T_{jj}}{\left(\sum_k T_{jk} + \sum_k T_{kj}\right) - T_{jj}} ,$$

can be used. The combination of all these options provides twelve alternative definitions for the constraint function that can be applied by the regionalisation algorithm (for SZ_{1-3} and SC_{1-2}), out of which six functions can be derived from:

$$\min\left\{1; \max\left[0; \alpha_7 + \frac{(1-\alpha_7)(\mathrm{SZ} - \beta_3)}{\beta_4 - \beta_3}\right]\right\} \min\left[1; \alpha_7 + \frac{(1-\alpha_7)(\mathrm{SC} - \beta_1)}{\beta_2 - \beta_1}\right] \geqslant \alpha_7 ,$$

and six from:

$$\mathrm{SC} - \frac{\alpha_8(\beta_4 - \beta_3)(\beta_2 - \beta_1)}{|\max(\beta_3 + 1; \mathrm{SZ}) - \beta_3|} \geqslant \beta_1 .$$

(3) Parameter α_8 is not used in the original algorithm by Coomber et al (1986), it is comprised in the objective functions proposed by this paper.

4.4 Plotting the resulting regions on a graph

It is very convenient to use the size and self-containment of the resulting regions provided by the regionalisation algorithm and the shape of the constraint function for further analyses. Each region can be plotted on an *x–y* graph as a point according to the values of its size and self-containment.[4] With regards to the notation of the constraint functions the regions will appear in the upper-right sector of the graph from the curve of the constraint function. Despite the fact that the algorithm and its application provide a relatively exact research method after the parameters have been set, the results depend significantly on the primary choice of the values of the parameters. Plotting the regions on the graph provides a refinement tool that can help to identify more suitable values for the parameters.

It is always better to start the testing with lower values of size and self-containment. Of course, the algorithm provides more regions during the first testing rounds, but their position on the *x–y* graph and knowledge of the settlement and regional system of a given territory can help the decision as to which regions should be eliminated by setting the size and self-containment parameters to higher values. Obviously this effect would not be achieved if the reverse procedure were undertaken, because the resulting graph would not include the regions that 'should' appear in the regionalisation. A graphical assessment of the results can help to identify the course and position of the constraint function on the graph. If there is a considerable gap in the field of points, this is the area of the discontinuity of the size and self-containment parameters or, more precisely, between the trade-off of these parameters.

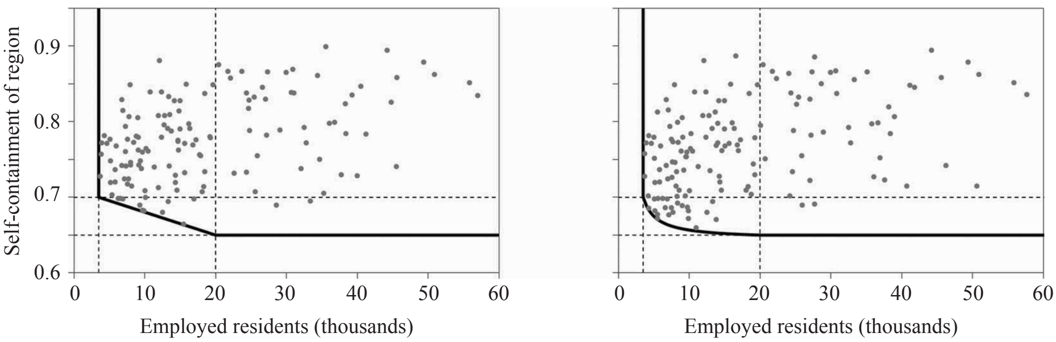
5 Example of application of the constraint function: the Czech Republic

The application of the regionalisation algorithm in the territory of the Czech Republic followed almost exactly the procedure proposed by Coombes et al (1986) in order to keep in line with the international literature on the issue. Thus the lower and upper size limits have been set to $\beta_3 = 3\,500$ and $\beta_4 = 20\,000$ employed residents ($\sum_k T_{jk}$). If the total self-containment of the regions had been used instead of the unidirectional self-containment, the lower and upper limits of the self-containment (β_1, β_2) would have been set to 0.65 and 0.70 (the original values would have been decreased by 0.05). The specifics of the Czech settlement system required further adjustments in the values of three parameters in order to achieve geographically relevant results based on thorough testing. Thus according to the original algorithm (Coombes et al, 1986) α_1 set to 0.80, α_2 to 0.50 and α_4 to 0.16. The crucial reason for these adjustments comes from the character of basic spatial units: that is the building blocks that were used for both regionalisations. The Czech municipalities (the lowest level for which the data on daily travel-to-work flows are available) are much larger basic spatial statistical units than British wards and they manifest a much greater degree of variability in terms of their size (for instance, the capital city of Praha is taken as one basic spatial unit, as is a village with only fifty economically active people). The original values for the α_1 and α_2 parameters (job ratio function and supply-side self-containment) had eliminated a considerable number of potential cores and the resulting regional patterns were completely unacceptable in geographical terms. These values were gradually lowered until they provided a sufficiently extensive set of potential cores. The original value of the α_4 parameter (0.10) meant that some distinct regional centres, that were able to form their own regions, had been amalgamated during the formation of multiple cores and could not have been dissolved in further steps. Extensive testing has proved that the value of 0.16 (on a scale ranging from 0.10 to 0.20) provided the greatest number of resulting regions—the inherent requirement for this type of regionalisation algorithm.

(4) A similar *x–y* graph of the size and self-containment of the regions has been used by Coombes and Casado-Díaz (2005) for the assessment of the development of the regional system.

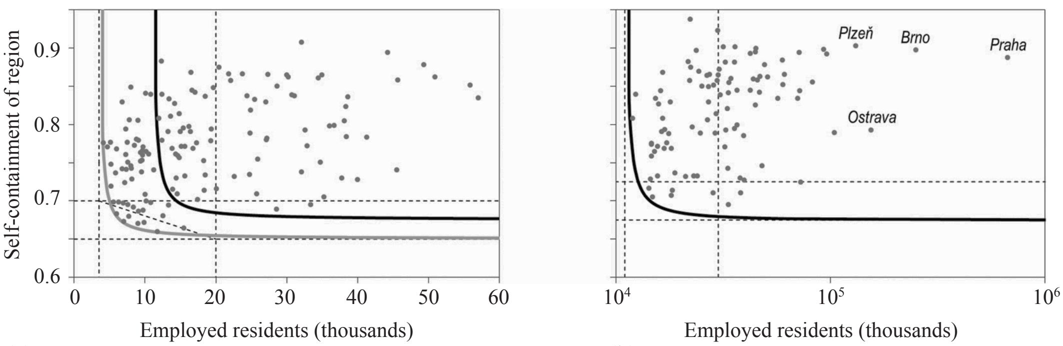
The constraint function has been slightly modified. The angled constraint function with $\alpha_7 = 0.90$ most closely resembled the original function and the algorithm produced 150 LLMAs [figure 4(a)]. With regards to the shape of the function, the largest number of LLMAs was produced by $\alpha_7 = 0.10$ [160 LLMAs: figure 4(b)]. The continuous constraint function with $\alpha_8 = 0.09$ produced a similar number of LLMAs to the first instance [151: figure 5(a)]. The difference between the first and the third cases [figure 4(a) compared with figure 5(a)] is in the regions that fulfil the conditions of the angled function and do not fulfil the conditions of the continuous function (four regions) and, conversely, in the regions that fulfil the continuous function and do not fulfil the angled function (five regions). Other differences between the two compared regional systems are minimal. Minor differences could have appeared in the assignment of municipalities, where the regions were dismembered and amalgamated iteratively, but they are generally statistically insignificant. The *x–y* graphs [figures 4 and 5(a)] present the size and self-containment of the regions. With regard to the character of the field of points and its illustration the twelve largest regions with more than 60 000 employed residents are missing.

In the next phase only the continuous constraint function was employed. If the aim was not to find the best possibility for delineating LLMAs, but to reach a local optimum in the value of the parameters entering the regionalisation algorithm (regardless of the size and self-containment of the regions), then an effort was made to place the curve of the constraint function onto the *x–y* graph, where there is a more significant gap in the field of points representing the regions. This graphical method provided the following values for the lower



(a) (b)

Figure 4. Position of local labour-market areas on the *x–y* graph with (a) angled constraint function; (b) nonlinear angled constraint function; both (a) and (b) lack the 12 largest regions with more than 60 000 employed residents.



(a) (b)

Figure 5. Position of local labour-market areas on the *x–y* graph with (a) continuous constraint function (without 12 largest regions with more than 60 000 employed residents; (b) shifted continuous constraint function (logarithmic scale).

and upper limits of the size and self-containment of the regions: $\beta_1 = 0.675$; $\beta_2 = 0.725$; $\beta_3 = 11\,000$; $\beta_4 = 30\,000$ (figure 5). The results of the procedures are presented in figure 5(b) (the logarithmic scale of the x-axis enables one to plot all regions, including the largest twelve) and in figure 6.

Figure 6. Local labour-market areas (LLMAs) of the Czech Republic (100 LLMAs based on the constraint function).

6 Discussion of the results

The shift of the curve of the constraint function has produced 100 LLMAs in the Czech Republic out of 6258 municipalities that are almost totally continuous (figure 6). There is an exception of six minor exclaves, each formed by one small municipality. Therefore these exclaves are not presented on the map (figure 6). The total self-containment of the regional system is 0.913, which means that 91.3% of all travel-to-work flows occur within the resulting LLMAs and only 8.7% of all travel-to-work flows cross the borders of the resulting LLMAs. Some remarks regarding the geographical relevance of the results should be made here. As the administrative division and some of the functional regionalisations of the Czech Republic (Klapka et al, 2013b; 2014; Maryáš and Řehák, 1987a; 1987b; Toušek et al, 2005) show, the number of regions at the microregional and mesoregional level varies between 70 and 200, depending on their purpose, function, and regionalisation method. The regional division provided by the second phase of the analysis carried out in this paper captures quite appropriately the spatial differentiation of the settlement and regional system of the Czech Republic. Apart from the regionalisation methods based on the first outgoing travel-to-work flow (see, eg, Halás et al, 2010), the regionalisation algorithm applied in this paper reduces, to a large extent, the regions organised around the largest cities (Praha, Brno). The interaction measure recommended by Smart (1974), that relativises and symmetrises the matrix of travel-to-work flows, produces a more realistic and well-arranged regional pattern of the LLMAs in the Czech Republic.

The spatial range of the regions of Praha and Brno remains rather large but still within the range of daily labour commuting rhythms. The same holds true for some other distinct regional centres, such as Plzeň, or České Budějovice, or even for some larger microregional centres (Mladá Boleslav, for example, with one of the largest employers in the Czech Republic). In contrast, the regional and microregional centres located in the agglomerations (Ostrava, Karviná, Frýdek-Místek in industrial northern Moravia and Silesia; Jablonec nad Nisou in northern Bohemia; Kladno in central Bohemia) or conurbations (Děčín, Ústí nad Labem, Teplice, Most, Chomutov in industrial northern Bohemia) manifest considerably lower levels of self-containment, while their size ranking is relatively high. In general, larger regions have higher levels of self-containment, with the obvious exception of the border regions, particularly those located in so-called promontories, where the state border makes up more than half of the regional borders (eg, Jeseník, Varnsdorf, Cheb). Despite their medium size of 15 000–30 000 economically active persons, these regions reach high levels of self-containment exceeding 0.9.

The results, that have been reached using the adjusted CURDS algorithm, can be viewed on two levels. The first level provides a comparison with the existing regionalisations of the Czech Republic based on daily travel-to-work flows. The second level provides a comparison with the results achieved by the application of the original version of the CURDS algorithm. In the latter case it is quite useful to point out the differences and advantages of the adjusted method used in the paper.

A comparison with older regionalisations based on the daily travel-to-work flows (eg, Maryáš and Řehák, 1987a; 1987b) shows the strengthened position of metropolitan areas (Praha, Brno) in the current regional system and the weakened and partially fragmented peripheral areas. More recent work based on newer data has also confirmed this trend. However, the Czech Republic has, so far, lacked the correct definition of functional regions, as the self-containment of the resulting regions has not been tested. Sometimes too many regions were delineated [eg, 260 regions identified by Halás et al (2010), when approximately one third of them did not meet the minimum 50% self-containment requirement], sometimes this requirement was not fulfilled even in the case of a more reasonable number of regions [eg, 260 regions identified by Halás et al (2010), when approximately one third of them did not meet the minimum 50% self-containment requirement]. However, the crucial insufficiency in these cases is the normative and unsupported estimation of the minimum size of regions, where there is no rule as to which regions should be maintained and which should be dissolved. Regardless of other parameters, the self-containment of 0.7 is fulfilled by 130–140 regions in the Czech regional system. However, in this instance a region can consist of a sole basic spatial unit (municipality) if its labour market is sufficiently self-contained (eg, mountain resorts). The shift of the constraint function and the application of Smart's measure (that slightly levels the size difference between regions) have provided a regional pattern of LLMAs of a comparable size that could also be used adequately as administrative units. These results also reflect the spatial extent of regional centres of the Czech Republic estimated by distance-decay functions based on daily travel-to-work flows (Halás et al, 2014).

The first difference between the results of the adjusted version and the original version of the regionalisation algorithm is caused by the different values of three α indexes. The reason is given above in this paper and the character of the regionalisation algorithm is not altered in any way. This step reflects only the specific character of the Czech regional and settlement system. All remaining adjustments to the regionalisation algorithm are related to the constraint function and their performance can be summarised in three points.

(1) The change from the use of the angled to the continuous constraint function is a dominant methodological contribution, because the trade-off between size and self-containment occurs along the whole curve and not only at a certain interval as in the original version.

The spatial patterns of the results from both the original and the adjusted algorithms do not differ significantly in this particular case [a comparison is given in figure 4, and 5(a)]. Differences occur in regions which are in the vicinity of the constraint function curve: that is, those that only slightly exceed the minimum levels of size and self-containment.

(2) The second contribution is both methodological and theoretical. The constraint function inherently comprises the index of self-containment. The proposed alternative to the index of total self-containment better reflects the requirement stated in the principal definition of a functional region than does the index of unidirectional self-containment: that is, a functional region has to be relatively self-contained with regard to certain movements of the population (in this particular case the daily travel-to-work flows). For instance, let us assume that the values for the index of unidirectional self-containment, that have been used up to now, are set in the interval 0.60–0.65. In this case, if the values for ingoing and outgoing flows are similar, it can mean that the minimum value of self-containment qualifying the functional region is not exceeded (at least 50% of all interactions incident to a region should occur within the region). Using the illustrative numbers, if the flows with their origins and destinations within a region (T_{jj}) make up 49% of all flows incident to the region, and outgoing (T_{jk}) and ingoing flows (T_{kj}) make up 25.5% each of all flows incident to the region, then the unidirectional self-containment is higher than 0.65, but the total self-containment is only 0.49.

(3) The third contribution is both methodological and practical. The proposed method of a shift of the constraint function in the *x–y* graph significantly helps the optimisation of the estimate for the minimum size and self-containment values for resulting regions and the trade-off between the two parameters. The advantage comes from the fact that the critical values for the size and self-containment parameters are not set normatively, but are estimated through an analysis of the spatial distribution of interactions within a particular regional and settlement system. This innovation also provides the opportunity to estimate the optimal size and self-containment criteria for regions in different territories: that is, in different regional and settlement systems. Finally it should be noted that the proposed continuous constraint function and its mathematical notation offer much more variability in the placement of the constraint function curve in the field of points in the *x–y* graph than the original angled constraint function and its mathematical notation.

7 Conclusion

The paper has the ambition to offer a methodological alternative to the rule-based procedures for the delineation of LLMAs in the definition and use of the constraint function. It addresses the issue of the size and self-containment of the regions, employing the daily travel-to-work flow data as the parameters entered into the regionalisation algorithm. In particular, the paper discusses the constraint function. The proposed formulation provides a mathematically elegant solution to the trade-off between size and self-containment. The trade-off is not compounded of several lines, but is expressed by a smooth continuous curve and its parameters can be easily changed without further mathematical adjustments.

The application of the CURDS algorithm (actually its second generation), with adjusted values of some parameters and steps in the regionalisation procedure and use of the proposed constraint function, has been tested in the territory of the Czech Republic. The resulting regions provide the expected regional pattern of the Czech Republic. Even without manual corrections of the results, the proposed LLMAs are compact and, with a few exceptions, contiguous. The resulting dominance of larger centres characterises the regular coexistence of Praha, Brno, Ostrava, and other regions with centres of around 100 000 inhabitants with the regions that lack a significantly dominant centre. The compactness of the regional system is preserved, even in areas with smaller microregional centres (for instance, along the northern part of the border between Bohemia and Moravia).

The paper puts forward several contributions in the field of functional regional taxonomy and particularly in the definition and use of the constraint function. First, similar to Casado-Díaz (2000) and unlike Coombes et al (1986), the four limits determining the size and self-containment of the resulting regions can be set in advance, and they are not dependent on the constraint function. Second, the constraint function uses an alternative definition of the self-containment parameter. Third, the trade-off between the size and the self-containment runs along the whole curve of the constraint function, not only between the lower and upper limits of the two parameters. Fourth, the proposed procedure and method are not dependent on the strict choice of the values of the input parameters, but, for instance, the lower and upper limits of the size and self-containment of the regions can be estimated graphically according to the position of the regions on the *x–y* graph. Although sophisticated mathematical methods could be applied to identify the best fit of the constraint function, this simple graphical solution enables the researchers to decide, according to their knowledge of the geographical reality, which regions near the curve of the constraint functions should be maintained and which dismembered. The size and self-containment parameters gained by the estimation procedure, the selected constraint function and the whole procedure, including plotting the information on an *x–y* graph, have a general validity and have not been adjusted for any particular territory (in this case for the Czech Republic). The proposed alternative and its use in the original CURDS algorithm can be applied generally and it is able to take into account the specifics of a given settlement and regional system, and correct the results quite successfully.

More general considerations which emerge from the findings of the paper can be put forward in the following way. If the demand that the methods of functional regional taxonomy should provide correct, functional regions (ie, those based on the aggregation of individual perceptions of geographic space and on the aggregation of the spatial behaviour and decisions of individuals) is taken for granted, then the application of the proposed constraint function and its further use promise to meet this demand. The estimation of the number of regions (eg, LLMAs), their boundaries, and spatial pattern is not based on normatively set criteria in this case, but in a certain way they reflect the spatial behaviour of individuals. Plotting the regions on the graph ensures that some aspects of this behaviour and its reflection in the spatial organisation are not lost in the resulting regional system and that two or more possible solutions to the region-building problem are well separated from one another in terms of the basic characteristics of the resulting regions (particularly the number of regions, their size, and self-containment). This forestalls such situations as two very similar regions being treated differently in the algorithm, where one will pass the rule set by the constraint function and will be considered to be an independent LLMA, while the other will be dissolved, as it narrowly does not meet the criterion. Such an approach to the analysis of the spatial behaviour (daily commuting to work in this case) appears to be better founded and more equitable in comparison with the normative estimate of parameters, because it better reflects the spatial aspects of human behaviour within a whole regional system. The use of the proposed constraint function as suggested in this paper ensures that the incorrect definition of LLMAs and the misinterpretation of spatial processes occurring both within and among LLMAs are avoided, or at least the risk is reduced.

If the LLMAs are based on the principle stated above, then they also better fulfil their applied function, such as planning or administrative regions. They are more appropriate for social economic analyses (particularly labour-market analyses), for assessment of population development and its redistribution, and for the identification of a regional dimension to settlement systems. This greater degree of applicability of LLMAs in comparison with administrative regions or normative regions is grounded in the fact that they are designed to minimise human mobility (in this case between the residence and workplace locations) and to maximise the profits rendered by the trade-off between the spatial aggregation and the spatial interaction as perceived by the individuals, because their decisions are responsible for the patterns of daily travel-to-work flows.

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